HYPER VERSION

Received: *November 1, 1998,* Accepted: *December 12, 1998*

# Induced dilaton in topologically massive quantum field theory*

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Abstract: We consider the conformally-invariant coupling of topologically massive gravity to a dynamical massless scalar field theory on a three-manifold with boundary. We show that, in the phase of spontaneously broken Lorentz and Weyl symmetries, this theory induces the target space zero mode of the vertex operator for the string dilaton field on the boundary of the three-dimensional manifold. By a further coupling to topologically massive gauge fields in the bulk, we demonstrate directly from the three-dimensional theory that this dilaton field transforms in the expected way under duality transformations so as to preserve the mass gaps in the spectra of the gauge and gravitational sectors of the quantum field theory. We show that this implies an intimate dynamical relationship between $T$-duality and $S$-duality transformations of the quantum string theory. The dilaton in this model couples bulk and worldsheet degrees of freedom to each other and generates a dynamical string coupling.

Keywords: Chern-Simons Theories, String Duality, Conformal Field Models in String Theory, 2D Gravity.

*This work was supported in part by the Particle physics and astronomy research council (U.K.)

## Contents





# 1. Introduction

The relationship between gravity in a three-dimensional spacetime and two-dimensional quantum gravity has been a topic of much interest over the last decade [1]–[8]. Of particular interest is topologically massive gravity [9] whose relation to Liouville theory [4, 5, 6, 8] extends the general correspondence between topological gauge theories in three dimensions and two-dimensional conformal field theories [10]. It also describes the gravitational sector of topological membrane theory [3, 11] (see [12] for a recent review) which reformulates string theory by filling in the string worldsheet and viewing it as the boundary of a three-manifold. Many aspects of string dynamics have intriguing interpretations when represented in terms of the dynamics of gauge and gravitational fields in the bulk. In this paper we will describe how to incorporate the dilaton field of string theory into this prescription.

In non-critical string theory, the target space tachyon operator sets the ultraviolet scale on the string worldsheet and it depends on the dilaton field $\phi$ and also the Liouville field. However, there is a dilatonic coupling in the string sigma-model action whose vacuum expectation value is proportional to the Euler character of the worldsheet and which thereby sets the string coupling constant $g_s$ according to

$$g_s = \left\langle \mathrm{e}^{\phi} \right\rangle \ . \tag{1.1}$$

One can change $g_s$ by shifting $\phi$ and thus naively spoil the conformal invariance of the theory at the quantum level. This also affects the tachyon operator, and hence the worldsheet scale, so that the dilaton field in this way controls the scale transformation properties of the string theory. This property is particularly important for the invariance of the string theory under duality transformations. If a direction of the $D$ dimensional target space is compactified on a circle of radius $R_0$, then $T$-duality maps this circle onto its dual of radius $R_0^* = \alpha'/R_0$, where $\alpha'$ is the string Regge slope. The expectation value of the dilaton must shift so as to leave unchanged the corresponding $D-1$ dimensional Planck scale,

$$\frac{R_0^*}{\kappa_{(D)}^{*\,2}} = \frac{R_0}{\kappa_{(D)}^2}\,, \tag{1.2}$$

where $\kappa_{(D)}$ is the $D$ dimensional gravitational coupling. For instance, for a non-linear sigma-model in a compact target space with metric $G$ that admits an isometry, the dilaton field is constant along the direction of the corresponding Killing vector and transforms under $T$-duality as [13]

$$\phi \to \phi^* = \phi + \frac{1}{2}\ln\frac{G_{00}}{\alpha'}\,, \tag{1.3}$$

where $G_{00} = R_0^2$ is the component of the metric tensor in the direction of the isometry generator.

On the other hand, the natural scale of the bulk gravitational theory is the three-dimensional Planck mass which sets the coupling constant of topologically massive gravity [14], and the induced worldsheet scale is set by the topological graviton mass [5]. This suggests that within this latter theory there could exist a three-dimensional version of the string dilaton field. Namely, the coupling of strings to a dilaton may be related to a three-dimensional gravitational model with a fluctuating Planck mass. In this paper we will show precisely how this is accomplished. We will consider the conformally-invariant coupling of topologically massive gravity to a dynamical, massless scalar field in the bulk [15]. In the phase of spontaneously broken Weyl invariance, the vacuum expectation value of the scalar field induces a mass for the graviton and gives a model reminescent of old theories of induced gravity [16]. When the model is written in a first-order (phase space) formalism for the gravitational fields [17], an induced two-dimensional $SL(2,\mathbb{R})$ gauged WZNW model emerges, in the phase of spontaneously broken $SO(2,1)$ gauge symmetry of the three-dimensional theory, which is well-known to be related to Liouville theory [18]. In the present framework, however, we shall find that the extra coupling to the scalar field induces a scale-dependent deformation of the usual Liouville theory which shares all of the properties of a (constant) dilaton term in a string sigma-model. In particular, by further coupling a bulk topologically massive gauge theory [9] to the scalar field (representing the inclusion of a string sigma-model action), we show that the requirement of invariance of the bulk physical spectrum of the quantum field theory under $T$-duality transformations of the associated target space leads



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to the usual form of the dilaton shift (1.3), in much the same way that the string theoretical requirement (1.2) does. In fact, as we will show, this correspondence suggests a remarkable equivalence between $T$-duality and $S$-duality transformations of the quantum string theory.

The origin of the dilaton field in string theory is somewhat mysterious, since in a sense it is really just a scaffold for producing the string coupling constant $g_s$. Although in the following we will not reproduce the complete dilaton vertex in the topological membrane picture, we do manage to capture that part of it which is essential for the qualitative features of the dilaton field in string theory. In particular the ensuing construction represents the proper incorporation of a string coupling into the topological membrane formulation of string theory, and the dilaton thereby induced by the bulk theory has a nice dynamical origin in terms of the geometry and the propagating particles in the three-manifold.

One crucial aspect of the deformed Liouville theory that we obtain is that it is *not* an intrinsically two-dimensional model. The breaking of the Lorentz and conformal symmetries of the three-dimensional quantum field theory induces propagating massive gauge and graviton degrees of freedom in the bulk. The massless scalar field couples bulk and boundary degrees of freedom in such a way that the three-dimensional dynamics controls the properties of the induced dilaton field. This immediately leads to the possibility of having a dilaton field which depends explicitly on the worldsheet coordinates and leads to a *dynamical* string coupling constant (1.1). The induced dilaton that we find is in fact related to the target space tachyon operator, so that in this case $g_s$ is a dynamical field in target space that controls the size of the spacetime. This is one of the basic features of 11-dimensional $M$-theory [19] whereby the string coupling constant of ten-dimensional type-IIA superstring theory is related to the radius of the eleventh dimension. In the present case we also find that the bulk dynamics induces a sort of new dimension into the model, in addition to the string embedding fields and the extra dimension induced by the Liouville field. This gives a potential dynamical origin for the extra dimension of spacetime inherent in $M$-theory from the basic point of view of fundamental string fields which could be relevant to the dynamics of the 11-dimensional theory itself.

The arrangement of this paper is as follows. In section 2, we describe the model whereby topologically massive gravity is conformally coupled to a scalar field theory and present the derivation of a "deformed" two-dimensional Liouville gravity induced on the boundary by the bulk action, following [5] for the most part. In section 3 we show that with an appropriate boundary condition on the scalar field one can reproduce the dilaton vertex operator on the boundary. In section 4 we consider the additional conformally invariant coupling to topologically massive gauge theories and derive the transformation laws (1.3) directly from the bulk theory. Finally, in section 5 we discuss the possibility of inducing a dynamical dilaton field on the worldsheet and its potential relevance to the dynamics of $M$-theory.

## 2. From conformally-coupled topologically massive gravity to deformed Liouville theory

In this section, we will consider the action for topologically massive gravity conformally coupled to a scalar field theory defined on a three-manifold with boundary. Following the derivation of two-dimensional quantum gravity from ordinary topologically massive gravity [4, 5, 6], we will derive a deformed Liouville theory induced on the two dimensional boundary by the bulk theory.

### 2.1 Definition of the bulk theory

Consider the action for topologically massive gravity defined on an orientable three-dimensional Minkowski-signature manifold $\mathcal{M}$ without boundary,

$$S_{TMG}[e,\omega] = \kappa \int_{\mathcal{M}} d^3x\ \epsilon^{\mu\nu\lambda} e_\mu^a R_{\nu\lambda}^a + \frac{k}{8\pi} \int_{\mathcal{M}} d^3x\ \epsilon^{\mu\nu\lambda} \left( \omega_\mu^a\, \partial_\nu \omega_\lambda^a + \frac{2}{3} \epsilon^{abc} \omega_\mu^a \omega_\nu^b \omega_\lambda^c \right), \tag{2.1}$$

where $\omega_\mu^a = \epsilon^{abc}\omega_\mu^{bc}$ is the $so(2,1)$ Lie algebra valued spin-connection and

$$R^a = d\omega^a + \epsilon^{abc}\omega^b \wedge \omega^c \tag{2.2}$$

is its curvature (we will use the differential form and component notations interchangeably). The first term in (2.1) is the Einstein-Hilbert action written in the first-order formalism, with $\kappa$ the three-dimensional Planck mass, while the second term is the gravitational Chern-Simons action. Unlike the first order Palatini action, the spin connection in (2.1) is not an independent variable but is related to the triads $e_\mu^a$ and the inverse triads $E_a^\nu$ by the Cartan-Maurer formula

$$\omega_\mu^a = \epsilon^{abc} \left[ E^{\nu\, b} \left( \partial_\mu e_\nu^c - \partial_\nu e_\mu^c \right) - \frac{1}{2}\, E^{\rho\, b} E^{\sigma\, c} \left( \partial_\rho e_\sigma^d - \partial_\sigma e_\rho^d \right) e_\mu^d \right], \tag{2.3}$$

with $e_\mu^a \otimes e_{\nu\, a} = g_{\mu\nu}$ and $E_\mu^a \otimes e_b^\mu = \delta_b^a$.

Consider a Weyl transformation of the metric of $\mathcal{M}$ (or equivalently a rescaling of the triad fields),

$$g_{\mu\nu}(x) \to \hat{g}_{\mu\nu}(x) = \Phi(x)^4 g_{\mu\nu}(x)\,, \qquad e_\mu^a(x) \to \hat{e}_\mu^a(x) = \Phi(x)^2 e_\mu^a(x) \tag{2.4}$$

where $\Phi(x)$ is some scalar field on $\mathcal{M}$, so that

$$\omega_\mu^a \to \hat{\omega}_\mu^a = \omega_\mu^a + \epsilon^{abc} E^{\nu b} e_\mu^c\, \partial_\nu \ln \Phi\,. \tag{2.5}$$

Under the transformations (2.4,2.5), the gravitational Chern-Simons action is invariant but the Einstein-Hilbert term changes, so that the total action (2.1) transforms as[1]

$$
\begin{aligned}
S_{TMG}[e,\omega] \to S_{TMG}[\hat{e},\hat{\omega}] &\equiv S_{CTMG}[e,\omega;\Phi] \\
&= \kappa \int_{\mathcal{M}} d^3x\ \epsilon^{\mu\nu\lambda}\,\Phi^2\, e^a_\mu R^a_{\nu\lambda} + \frac{k}{8\pi}\int_{\mathcal{M}} d^3x\ \epsilon^{\mu\nu\lambda}\left(\omega^a_\mu\,\partial_\nu\omega^a_\lambda + \frac{2}{3}\epsilon^{abc}\omega^a_\mu\omega^b_\nu\omega^c_\lambda\right) \\
&\quad + 8\kappa\int_{\mathcal{M}} d^3x\ \sqrt{g}\ g^{\mu\nu}\,\partial_\mu\Phi\,\partial_\nu\Phi\,. 
\end{aligned}
\tag{2.6}
$$

In the naive vacuum $\langle\Phi^2\rangle = 0$, the spectrum of the quantum field theory (2.6) contains a massless scalar particle but no graviton degrees of freedom, so that in this phase the model is equivalent to the pure conformally-invariant gravitational Chern-Simons theory which is a topological $SO(2,1)$ gauge theory. A more interesting case is when the conformal symmetry of the pure Chern-Simons action is spontaneously broken and induces the Einstein-Hilbert term by a sort of Higgs mechanism [15, 16]. If the scalar field $\Phi(x)^2$ has a non-zero vacuum expectation value, then one can gauge it away by a Weyl transformation (2.4) with conformal factor $\Omega(x)^2 = \langle\Phi^2\rangle/\Phi(x)^2$. Then, with the rescaling $\Phi \to \Omega\,\Phi$, (2.6) becomes the topologically massive gravity action (2.1) for the fields $\hat{e},\hat{\omega}$ with Planck mass $\hat{\kappa} = \kappa\,\langle\Phi^2\rangle$. This shows that the Hartree-Fock average of the fluctuating field $\Phi$ is related to the topological graviton mass $M_g$ via

$$
M_g = \frac{8\pi\kappa\,\langle\Phi^2\rangle}{k}\,. \tag{2.7}
$$

In other words, in the theory (2.6) the background field $\Phi$ (or rather its vacuum expectation value generated by zero-point quantum fluctuations) sets the mass scale of the bulk theory. Since the perturbation expansion parameter of topologically massive gravity is the super-renormalizable coupling constant $M_g/\kappa = 8\pi\langle\Phi^2\rangle/k$ [14], it follows that $\Phi$ also determines the effective coupling constant for the model (2.6).[2] This parallels the case in string theory where the vacuum expectation value of the dilaton field sets the string coupling constant (1.1). In what follows we will make this correspondence more precise.

The action (2.6) is invariant under the conformal transformations

$$
\begin{aligned}
\Phi(x) &\to \Omega(x)\,\Phi(x)\,, \\
g_{\mu\nu}(x) &\to \Omega(x)^{-4}\,g_{\mu\nu}(x)\,, \\
e^a_\mu(x) &\to \Omega(x)^{-2}\,e^a_\mu(x)\,, \\
\omega^a_\mu(x) &\to \omega^a_\mu(x) + \epsilon^{abc}E^{\nu b}(x)e^c_\mu(x)\,\partial_\nu\ln\Omega(x)\,,
\end{aligned}
\tag{2.8}
$$

[1]Note that one could also include a cosmological constant term in the action (2.1). Under the conformal transformation (2.4) this term would induce a $\Phi^6$ potential in the action (2.6). This situation is relevant to the corresponding construction for Einstein gravity on $AdS_3$ spacetimes which also induces Liouville theory [7]. We shall not discuss this aspect in this paper.

[2]Note that we can absorb the mass scale $\kappa$ into the field $\chi = 4\sqrt{\kappa}\,\Phi$ so that $\chi$ has the correct canonical dimension $\frac{1}{2}$ for a bosonic field in three dimensions.

and we shall refer to the model (2.6) as conformally-coupled topologically massive gravity. However, if $\mathcal{M}$ has a non-empty boundary $\partial\mathcal{M}$, then the conformal symmetry of the pure bulk action (2.6) is explicitly broken. Under the local scale transformation (2.4), the Einstein-Hilbert part of the action induces an extra boundary term,

$$S_{TMG}[e,\omega] \to S_{CTMG}[e,\omega;\Phi] - 8\kappa \oint_{\partial\mathcal{M}} \Phi\,\partial_\perp\Phi\,, \tag{2.9}$$

where $\partial_\perp$ denotes the normal derivative to the boundary of $\mathcal{M}$. To eliminate this term from the action one would have to impose a Neumann boundary condition for the field $\Phi$ on $\partial\mathcal{M}$. This will be discussed in section 5. Here we shall impose a Dirichlet boundary condition on the scalar field,

$$t^\alpha\,\partial_\alpha\Phi = 0 \qquad \text{or} \quad \Phi(x)\,\Big|_{\partial\mathcal{M}} = \text{constant}\,, \tag{2.10}$$

where $t^\alpha$ is a unit vector along the boundary $\partial\mathcal{M}$. The choice of boundary condition (2.10) will ensure that the induced dilaton field that arises is not an explicit function of the worldsheet coordinates (but generally only an implicit one through its dependence on the target space fields and the Liouville field). As a consequence of the Dirichlet boundary condition, the full conformal symmetry group of the three-dimensional theory is broken down to the subgroup of conformal transformations (2.8) which are constant on the boundary of $\mathcal{M}$. Thus the conformal symmetry group of conformally-coupled topologically massive gravity will induce the group of global scale transformations of the induced two-dimensional field theory.

### 2.2 Derivation of the induced boundary theory

The action for conformally-coupled topologically massive gravity can be written in the form

$$\begin{aligned} S_{CTMG}[e,\omega,\Phi,\lambda] = \kappa\int_{\mathcal{M}}\Phi^2\, e^a\wedge R^a + \frac{k}{8\pi}\int_{\mathcal{M}}\left(\omega^a\wedge d\omega^a + \frac{2}{3}\epsilon^{abc}\omega^a\wedge\omega^b\wedge\omega^c\right) + \\ +8\kappa\int_{\mathcal{M}}(d\Phi)^2 - \; 8\kappa\oint_{\partial\mathcal{M}}\Phi\,\partial_\perp\Phi + \\ +\int_{\mathcal{M}}\Phi^2\,\lambda^a\wedge\left(de^a + \epsilon^{abc}\omega^b\wedge e^c + \epsilon^{abc}e^b\wedge\omega^c\right), \end{aligned} \tag{2.11}$$

where the spin-connection $\omega$ and the triad field $e$ are to be treated as independent variables. Here $\lambda$ is a Lagrange multiplier field that enforces the torsion-free constraint (2.3) on the geometry and which is invariant under the Weyl transformations (2.8). The generally-covariant action (2.11) is invariant under the restricted conformal transformations (2.8) and also under the local $SO(2,1)$ Lorentz transformations whose infinitesimal forms are

$$\begin{aligned} \delta_\theta e^a &= \epsilon^{abc}\theta^b e^c\,, & \delta_\theta\lambda^a &= \epsilon^{abc}\theta^b\lambda^c\,, \\ \delta_\theta\omega^a &= -\tfrac{1}{2}\left(d\theta^a + 2\epsilon^{abc}\theta^b\omega^c\right)\,, & \delta_\theta\Phi &= 0\,. \end{aligned} \tag{2.12}$$

Note that one can rewrite (2.11) up to an overall constant as an $SL(2,\mathbb{R})$ gauge theory by simply rescaling the action according to the trace relationship $\mathrm{Tr}(T_a T_b) = 4\,\mathrm{tr}(\tau_a \tau_b)$, where $T_a$ and $\tau_a$ are the generators of the fundamental representations of $SO(2,1)$ and $SL(2,\mathbb{R})$, respectively.

If $\partial\mathcal{M} \neq \emptyset$, then one needs to augment the action with appropriate boundary terms to ensure that the resulting path integral formulation of the quantum field theory has a semi-classical approximation [4, 5, 20] (this is equivalent to selecting the necessary boundary conditions to solve the field equations). After integrating the Einstein-Hilbert term by parts, the action (2.11) can be rewritten as

$$S_{CTMG}[e,\omega,\Phi,\beta] = S_{\text{bulk}}[e,\omega,\Phi,\beta] + \kappa \oint_{\partial\mathcal{M}} \Phi^2\ \mathrm{tr}\,(\omega \wedge e) - 2\kappa \oint_{\partial\mathcal{M}} \Phi\,\partial_\perp \Phi\,, \quad (2.13)$$

where

$$S_{\text{bulk}}[e,\omega,\Phi,\beta] = \int_{\mathcal{M}} \mathrm{tr}\left[\Phi^2 \beta \wedge (de + \omega\wedge e - e\wedge\omega) + \frac{k}{8\pi}\left(\omega\wedge d\omega + \frac{2}{3}\omega\wedge\omega\wedge\omega\right)\right]$$
$$+\,\kappa \int_{\mathcal{M}} \mathrm{tr}\Big[\Phi\, e\wedge\omega\wedge(2\,d\Phi + \Phi\,\omega)\Big] + 2\kappa \int_{\mathcal{M}} (d\Phi)^2 \quad (2.14)$$

is the contribution from the bulk parts of the fields, and the field

$$\beta^a = \lambda^a + \kappa\,\omega^a \quad (2.15)$$

transforms like the spin-connection under both Lorentz and Weyl transformations. Under variations of the fields which do not necessarily vanish on the boundary, the variation of the action (2.13), restricted to the boundary, is given by

$$\delta S_{CTMG}[e,\omega,\Phi,\beta]\,\Big|_{\partial\mathcal{M}} = -\oint_{\partial\mathcal{M}} \mathrm{tr}\left[\Phi^2\beta\wedge\delta e + \frac{k}{8\pi}\,\omega\wedge\delta\omega - \kappa\Phi^2(\delta\omega\wedge e + \omega\wedge\delta e)\right]. \quad (2.16)$$

Note that the field $\Phi$ is not varied on the boundary due to the Dirichlet boundary condition (2.9). Eq. (2.16) shows that we need to add appropriate surface terms to the action to cancel the boundary variations of the various fields. The precise form of these terms depends on the boundary conditions imposed on the various phase space variables. Choosing a complex structure on $\partial\mathcal{M}$, from (2.16) it follows that there are three sets of canonical pairs, namely $(\beta_z, e_{\bar z})$, $(\beta_{\bar z}, e_z)$ and $(\omega_z, \omega_{\bar z})$. Thus, we need to specify boundary conditions on one of the components from each canonical pair.

Accordingly, let us make the following choice of boundary conditions on $\partial\mathcal{M}$,

$$\delta e_z = \delta\beta_z = \delta\omega_{\bar z} = 0\,. \quad (2.17)$$

With these boundary conditions the terms required to be added to the action are

$$S_B[e,\omega,\Phi,\beta] = -\oint_{\partial\mathcal{M}} d^2z\ \mathrm{tr}\left[\frac{k}{8\pi}\,\omega_z\omega_{\bar z} - \Phi^2 e_{\bar z}\,(\beta_z - \kappa\omega_z)\right], \quad (2.18)$$

so that the total action reads

$$
\begin{aligned}
S_T[e,\omega,\Phi,\beta] &= S_{CTMG}[e,\omega,\Phi,\beta] + S_B[e,\omega,\Phi,\beta] \\
&= S_{\text{bulk}}[e,\omega,\Phi,\beta] - 2\kappa \oint_{\partial\mathcal{M}} \Phi\, \partial_\perp \Phi - \\
&\quad - \oint_{\partial\mathcal{M}} d^2z \ \mathrm{tr}\left[\frac{k}{8\pi}\,\omega_z\omega_{\bar z} - \Phi^2\left(e_{\bar z}\beta_z - \kappa e_z\omega_{\bar z}\right)\right].
\end{aligned}
\tag{2.19}
$$

Under the $SL(2,\mathbb{R})$ gauge transformations

$$
\begin{aligned}
e &\to g^{-1}eg\,, \\
\beta &\to g^{-1}(d+\beta)g\,, \\
\omega &\to g^{-1}(d+\omega)g\,, \\
\Phi &\to \Phi\,,
\end{aligned}
\tag{2.20}
$$

the action (2.19) is not invariant because the Chern-Simons term in (2.14) and the $\omega_z\omega_{\bar z}$ term in (2.19) both induce additional boundary terms [10]. Under the transformations (2.20) a chiral gauged $SL(2,\mathbb{R})$ WZNW model is induced on the boundary. The resulting action is given by

$$
\begin{aligned}
S_T[e,\omega,\Phi,\beta] \to S_T^-[g;e,\omega,\Phi,\beta] &= S_T[e,\omega,\Phi,\beta] + \\
&\quad + \frac{k}{8\pi}\oint_{\partial\mathcal{M}} d^2z \ \mathrm{tr}\left[(\partial_z g)g^{-1}(\partial_{\bar z} g)g^{-1} + 2(\partial_z g)g^{-1}\omega_{\bar z}\right] + \\
&\quad + \frac{k}{24\pi}\int_{\mathcal{M}} \mathrm{tr}\left(g^{-1}dg\wedge g^{-1}dg\wedge g^{-1}dg\right).
\end{aligned}
\tag{2.21}
$$

Note that although the WZNW term in (2.21) appears to be supported on the bulk manifold $\mathcal{M}$, it actually turns out to be a total derivative and can thus be written as a boundary term for the group elements $g(z,\bar z) \in SL(2,\mathbb{R})$.

The action (2.21) is invariant under the left $SL(2,\mathbb{R})$ gauge transformations

$$
\begin{aligned}
g &\to hg\,, \\
e &\to heh^{-1}\,, \\
\beta &\to h(d+\beta)h^{-1}\,, \\
\omega &\to h(d+\omega)h^{-1}\,, \\
\Phi &\to \Phi\,.
\end{aligned}
\tag{2.22}
$$

We shall consider here the perturbative phase of the theory in which there is a non-zero condensate of the dreibeins, i.e. $\langle e^a_\mu\rangle = \rho\,\delta^a_\mu$, where $\rho \neq 0$ is a conformal factor. In this phase the local $SL(2,\mathbb{R})$ gauge symmetry (2.22) is spontaneously broken, and we can fix the pullback of the dreibein component $e_z$ to the boundary to be

$$
e_z = \begin{pmatrix} 0 & \rho \\ 0 & 0 \end{pmatrix}.
\tag{2.23}
$$

If we consider another copy of $\partial\mathcal{M}$ of the opposite chirality and orientation to that appearing above, and fix $e_{\bar z} = e_z^\top$ on the opposite boundary, then the total induced metric on the chirally-symmetric boundary is $g_{z\bar z} = \mathrm{tr}(e_z e_{\bar z}) = \rho^2$. In the topological phase $\langle e_\mu^a \rangle = 0$, there is no background spacetime and again there are no local graviton degrees of freedom. From the Cartan-Maurer equation (2.3) it follows that the fixed component of the spin-connection on $\partial\mathcal{M}$ corresponding to the above choice of dreibein is

$$\omega_{\bar z} = \begin{pmatrix} \partial_{\bar z}\rho & 0 \\ 0 & -\,\partial_{\bar z}\rho \end{pmatrix} . \tag{2.24}$$

However, we shall need to keep all fields arbitrary as yet until we carry out a proper gauge-fixing of the path integral.

Note that the boundary term (2.18), added to the conformally-coupled topologically massive gravity action in (2.13), involves only the component $e_{\bar z}$ of the dreibein field on $\partial\mathcal{M}$. This implies that, despite the gauge choice (2.23), there is still a residual abelian gauge symmetry in (2.21) defined by the action of $h$ in (2.22) which restricted to the boundary lies in the Borel subgroup $\mathcal{B}_-$ of the total $SL(2,\mathbb{R})$ group consisting of lower triangular matrices,

$$h = \begin{pmatrix} 1 & 0 \\ \xi & 1 \end{pmatrix} . \tag{2.25}$$

Let us now consider a local Gauss decomposition of the matrix-valued field $g$ in terms of the two Borel subgroups $\mathcal{B}_\pm$ and the Cartan subgroup of $SL(2,\mathbb{R})$,

$$g = \begin{pmatrix} 1 & 0 \\ \xi & 1 \end{pmatrix} \begin{pmatrix} \mathrm{e}^{-\varphi} & 0 \\ 0 & \mathrm{e}^{\varphi} \end{pmatrix} \begin{pmatrix} 1 & \psi \\ 0 & 1 \end{pmatrix} . \tag{2.26}$$

In terms of the variables in (2.26) and the decomposition

$$\omega_{z,\bar z} = \begin{pmatrix} \omega^3_{z,\bar z} & \omega^+_{z,\bar z} \\ \omega^-_{z,\bar z} & -\,\omega^3_{z,\bar z} \end{pmatrix} \tag{2.27}$$

of the spin-connection, the action (2.21) can be evaluated using the Polyakov-Wiegmann identity [21] for the WZNW action evaluated on the product of the three matrix-valued fields in (2.26). The resulting action is invariant under the local residual Borel subgroup symmetry of the holomorphic part of the dreibein condensate,

$$\begin{aligned} h &\to h + \tau_-\theta \\ \omega_{\bar z} &\to \omega_{\bar z} - \tau_-\,\partial_{\bar z}\theta \,, \end{aligned} \tag{2.28}$$

where

$$\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} , \qquad \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} , \qquad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -\,1 \end{pmatrix} , \tag{2.29}$$



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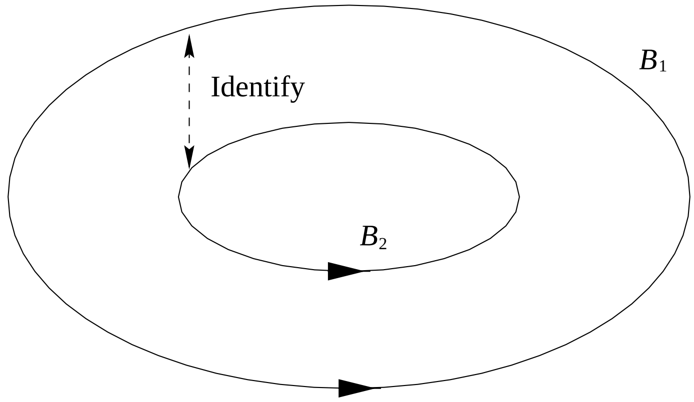


**Figure 1:** The annular spatial slice, with two boundaries $B_1 = \partial\mathcal{M}_+$ and $B_2 = \partial\mathcal{M}_-$, corresponding to the three-geometry $\mathcal{M} = \Sigma \times \mathbb{R}$ where $\Sigma$ is an annulus. Note that the boundaries are oriented oppositely so that they can be glued together.

are the generators of the fundamental representation of $SL(2,\mathbb{R})$, and is given by

$$\begin{aligned} S_T^-[g;e,\omega,\Phi,\beta] = S_{\rm bulk}[e,\omega,\Phi,\beta] - 2\kappa \oint_{\partial\mathcal{M}} \Phi\, \partial_\perp \Phi + \frac{k}{4\pi} \oint_{\partial\mathcal{M}} d^2z\ \partial_z\varphi\, \partial_{\bar z}\varphi - \\ -\frac{k}{8\pi} \oint_{\partial\mathcal{M}} d^2z\ \left[2\omega_z^a\omega_{\bar z}^a - 4\omega_{\bar z}^3\, \partial_z\varphi + \omega_{\bar z}^- \left({\rm e}^{-2\varphi}\, \partial_z\psi + \frac{8\pi\kappa}{k}\, \rho\, \Phi^2\right)\right]. \end{aligned} \tag{2.30}$$

In (2.31) we have cancelled the boundary $\beta_z e_{\bar z}$ term in (2.19) with the gauge dependence of the $\omega_{\bar z} e_z$ term and used the extra abelian $\mathcal{B}_-$ gauge symmetry (2.28) to select the gauge $\xi = 0$. Having fixed the gauge, we also set the dreibein components to their fixed boundary values (2.23).

However, our construction thus far produces only one chiral sector of the worldsheet theory on $\partial\mathcal{M}$. The full chirally-symmetric theory is obtained by taking the geometry of the manifold $\mathcal{M}$ to be such that its boundary is the connected sum $\partial\mathcal{M} = \partial\mathcal{M}_+ \,\#\, \partial\mathcal{M}_-$ of two isomorphic surfaces $\partial\mathcal{M}_\pm$ of opposite chirality and orientation (see fig. 1)[3] and gauging the action with respect to both the lower Borel subgroup $\mathcal{B}_-$ for the left-moving sector and the upper Borel subgroup $\mathcal{B}_+$ for the right-moving sector [18].

If, say, the action (2.31) is defined on $\partial\mathcal{M}_-$, then we can derive its anti-holomorphic counterpart on $\partial\mathcal{M}_+$ in the same manner as above by choosing the opposite chiral components in the boundary conditions (2.17), inducing an anti-chiral gauged WZNW action which is invariant under right $SL(2,\mathbb{R})$ gauge transformations, and thereby producing a boundary action which is invariant under the residual $\mathcal{B}_+$ symmetry of the anti-holomorphic part of the dreibein condensate. Taking into careful account of the change of orientation on $\partial\mathcal{M}_+$ (fig. 1), this leads to the anti-chiral

[3] For example, we could take $\mathcal{M} = \Sigma \times [0,1]$ where $\Sigma \times \{0\}$ and $\Sigma \times \{1\}$ are surfaces of opposite chirality and orientation.

version of (2.31),

$$
\begin{aligned}
S_T^{+}[g;e,\omega,\Phi,\beta] = {}& S_{\rm bulk}[e,\omega,\Phi,\beta] + 2\kappa\oint_{\partial\mathcal{M}}\Phi\,\partial_\perp\Phi - \frac{k}{4\pi}\oint_{\partial\mathcal{M}} d^2z\;\partial_z\varphi\,\partial_{\bar z}\varphi \\
&+\frac{k}{8\pi}\oint_{\partial\mathcal{M}} d^2z\;\left[2\omega_z^a\omega_{\bar z}^a - 4\omega_z^3\,\partial_{\bar z}\varphi + \omega_z^+\left(\mathrm{e}^{-2\varphi}\,\partial_{\bar z}\xi + \frac{8\pi\kappa}{k}\,\rho\,\Phi^2\right)\right]
\end{aligned}
\tag{2.31}
$$

in the gauge $\psi = 0$. Gluing the two contributions (2.31) and (2.32) together [18], we arrive in this way at the full chirally symmetric action

$$
\begin{aligned}
S_T^{\rm sym}[g;e,\omega,\Phi,\beta] = {}& S_{\rm bulk}[e,\omega,\Phi,\beta] - 2\kappa\oint_{\partial\mathcal{M}}\Phi\,\partial_\perp\Phi + \\
&+\frac{k}{8\pi}\oint_{\partial\mathcal{M}} d^2z\;\Big[2\,\partial_z\varphi\,\partial_{\bar z}\varphi + \,\mathrm{e}^{-2\varphi}\,\omega_z^+\omega_{\bar z}^- - \omega_z^-\omega_{\bar z}^+ - \\
&\qquad -\frac{8\pi\kappa}{k}\,\rho\,\Phi^2\left(\omega_z^+ + \omega_{\bar z}^-\right) - \\
&\qquad -2\left(\omega_z^3\omega_{\bar z}^3 - 2\omega_z^3\,\partial_{\bar z}\varphi - 2\omega_{\bar z}^3\,\partial_z\varphi\right)\Big]\,,
\end{aligned}
\tag{2.32}
$$

where we have used the remnant abelian $\mathcal{B}_+\times\mathcal{B}_-$ gauge symmetry to fix the gauge $\xi = \psi = 0$ on $\partial\mathcal{M}$. The appearence of the $\mathrm{e}^{-2\varphi}\omega_z^+\omega_{\bar z}^-$ term in (2.32) comes from the appropriate gluing required to produce the $\mathcal{B}_+\times\mathcal{B}_-$ symmetric WZNW model [18]. This equivalence follows from the fact [1, 4, 8] that the boundary dynamics of ordinary topologically massive gravity induce the full chirally symmetric Liouville theory.

The partition function of conformally-coupled topologically massive gravity is defined by the gauge-fixed path integral

$$
\begin{aligned}
\mathcal{Z} = {}& \int[de]\;\Delta_{\rm FP}[e]\;[d\beta]\;\Delta_{\rm FP}[\beta]\;[d\omega]\;\Delta_{\rm FP}[\omega]\;\times \\
&\times\int[d\Phi]\;\int[dg]\;\delta(\mathcal{E}[\bar e])\;\delta(\mathcal{L}[{}^g\beta])\;\delta(\mathcal{W}[{}^g\omega])\;\mathrm{e}^{iS_{CTMG}[e,\omega,\Phi,\beta]}
\end{aligned}
\tag{2.33}
$$

where ${}^g\beta = g^{-1}\beta g + g^{-1}dg$ and $\bar e = g^{-1}eg$. Here $\Delta_{\rm FP}$ denotes the Faddeev-Popov determinant, $\mathcal{E}$, $\mathcal{L}$ and $\mathcal{W}$ are gauge-fixing functions, and $dg$ is the left-right invariant Haar measure on $SL(2,\mathbb{R})$. Following the steps which led to the effective action (2.32), we see that (2.33) can be written as

$$
\begin{aligned}
\mathcal{Z} = {}& \mathcal{N}\int[d\bar e]\;\delta(\mathcal{E}[\bar e])\;\Delta_{\rm FP}[\bar e]\;[d\bar\beta]\;\delta(\mathcal{L}[\bar\beta])\;\Delta_{\rm FP}[\bar\beta]\;[d\bar\omega]\;\delta(\mathcal{W}[\bar\omega])\;\Delta_{\rm FP}[\bar\omega]\times \\
&\times\int[d\Phi]\mathrm{e}^{iS_{\rm bulk}[\bar e,\bar\omega,\Phi,\bar\beta]}\int[d\varphi]\,\det[\partial_z\,\partial_{\bar z}]\,\exp\left\{i\oint_{\partial\mathcal{M}}\left[\frac{k}{4\pi}\,d^2z\partial_z\varphi\,\partial_{\bar z}\varphi - 2\kappa\,\Phi\,\partial_\perp\Phi\right]\right\}\times \\
&\times\int[d\omega_z^+][d\omega_{\bar z}^-]\,\exp\left\{-\frac{ik}{8\pi}\oint_{\partial\mathcal{M}} d^2z\left[\mathrm{e}^{-2\varphi}\,\omega_z^+\omega_{\bar z}^- + \frac{8\pi\kappa}{k}\,\rho\,\Phi^2\left(\omega_z^+ + \omega_{\bar z}^-\right)\right]\right\}\times \\
&\times\int[d\omega_z^3][d\omega_{\bar z}^3]\,\exp\left\{-\frac{ik}{4\pi}\oint_{\partial\mathcal{M}} d^2z\left[\omega_z^3\omega_{\bar z}^3 - 2\omega_z^3\,\partial_{\bar z}\varphi - 2\omega_{\bar z}^3\,\partial_z\varphi\right]\right\},
\end{aligned}
\tag{2.34}
$$

where the bars on the fields denote their bulk values which are parametrized by their adjoint orbits under the $SL(2,\mathbb{R})$ gauge group in (2.20), and the additional determinant comes from gauge-fixing the $\mathcal{B}_+ \times \mathcal{B}_-$ Borel symmetry. Here and in the following we will absorb irrelevant (infinite) constants into the normalization factor $\mathcal{N}$. The functional integration over the boundary spin-connection components in (2.34) is Gaussian and yields a fluctuation determinant that can be evaluated to give [22]

$$\prod_{(z,\bar{z})\in\partial\mathcal{M}} \mathrm{e}^{2\varphi(z,\bar{z})} = \mathcal{N}\exp\frac{i}{8\pi}\oint_{\partial\mathcal{M}} d^2z\ Q\,\varphi\,R^{(2)}\,, \tag{2.35}$$

where $R^{(2)}$ is the worldsheet scalar curvature of $\partial\mathcal{M}$. The constant $Q$ in (2.35) is a regularization parameter which will control the central charge of the induced Liouville theory. Upon rescaling $\varphi \to \varphi/\sqrt{10k}$ and $Q \to \sqrt{10k}\,Q$ we arrive finally at

$$\begin{aligned}\mathcal{Z} = \mathcal{N}\int[d\bar{e}]\ \delta(\mathcal{E}[\bar{e}])\ \Delta_{\rm FP}[\bar{e}]\ [d\bar{\beta}]\ \delta(\mathcal{L}[\bar{\beta}])\ \Delta_{\rm FP}[\bar{\beta}]\ [d\bar{\omega}]\ \delta(\mathcal{W}[\bar{\omega}])\ \Delta_{\rm FP}[\bar{\omega}]\\ \times\int[d\Phi]\ \mathrm{e}^{iS_{\rm bulk}[\bar{e},\bar{\omega},\Phi,\bar{\beta}]}\ \int[d\varphi]\ \mathrm{e}^{iS_\partial[\varphi;\Phi]}\,,\end{aligned} \tag{2.36}$$

where

$$S_\partial[\varphi;\Phi] = \oint_{\partial\mathcal{M}} d^2z\left(\frac{1}{8\pi}\,\partial_z\varphi\,\partial_{\bar{z}}\varphi + \frac{Q}{8\pi}\,\varphi\,R^{(2)} + \frac{2\pi\kappa^2}{k}\,\rho^2\,\Phi^4\,\mathrm{e}^{\sqrt{2/5k}\,\varphi}\right) - 2\kappa\oint_{\partial\mathcal{M}}\Phi\,\partial_\perp\Phi\,. \tag{2.37}$$

Note that the dreibein condensate parameter $\rho^2 = \sqrt{g}$ cancels in the first two terms of (2.37) because of the additional contractions with the induced metric $g^{z\bar{z}}$ required for worldsheet general covariance.

The boundary induced action (2.37) is very similar to that for two-dimensional quantum gravity, with the field $\varphi$ which parametrizes the Cartan subgroup of the three-dimensional Lorentz symmetry group $SL(2,\mathbb{R})$ identified as the Liouville field. However, there are two crucial differences. The first one is that although the three-dimensional scalar field $\Phi$ is constant on $\partial\mathcal{M}$, it is scale-dependent, and it therefore defines a dynamical conformal deformation of the cosmological constant operator in (2.37). The vacuum expectation value of the scalar field $\Phi$ determines the two-dimensional cosmological constant $\mu$, i.e. the scale of Liouville theory, which with the normalization in (2.37) is given by

$$\mu = \frac{1}{10}M_g^2 \tag{2.38}$$

where $M_g$ is the topological graviton mass (2.7). Thus the natural scale of the two-dimensional boundary theory is determined by that of the bulk three-dimensional theory [5], so that the critical value $\mu = 0$ comes from unbroken $SO(2,1)$ conformal symmetry of the three-dimensional dynamics. The second difference is that the additional boundary term in (2.37) which is independent of the Liouville field depends on the bulk value of $\Phi$ in a neighbourhood of the boundary $\partial\mathcal{M}$. This term does not affect the Weyl transformation properties of the induced two-dimensional theory and merely serves to maintain the conformal symmetry of the bulk part of the three-dimensional theory (2.36). In particular, it prevents the complete factorization of bulk and boundary degrees of freedom in the quantum field theory (in contrast to the usual cases [4, 10]), so that the theory (2.37) is intrinsically three-dimensional in origin. This feature is important to remember when analysing certain aspects such as the conformal invariance properties of the boundary theory. We shall refer to the model defined by the action (2.37) as "deformed Liouville theory".

## 3. Induced dilaton field and conformal symmetry

In the previous section we have shown that, in the phase with spontaneous breaking of the conformal and Lorentz symmetries of the three-dimensional quantum field theory, conformally-coupled topologically massive gravity induces a deformed Liouville theory (2.37) on the boundary of the three-dimensional spacetime. The dynamical scaling constant $\Phi|_{\partial\mathcal{M}}$ can be removed from the cosmological constant term in (2.37) by the shift $\varphi \to \varphi - \frac{1}{\alpha_+} \ln \Phi^4$ of the Liouville field, where $\alpha_+ = \sqrt{\frac{2}{5k}}$, so that the action (2.37) can be written as

$$S_\partial[\varphi;\Phi] = \oint_{\partial\mathcal{M}} d^2z \left( \frac{1}{8\pi}\, \partial_z\varphi\, \partial_{\bar z}\varphi + \frac{Q}{8\pi}\, \varphi\, R^{(2)} + \frac{\rho^2\, \hat\mu}{8\pi\alpha_+^2}\, \mathrm{e}^{\alpha_+\varphi} \right) - \\ - \frac{Q}{\alpha_+}\, S_D[\Phi] - 2\kappa \oint_{\partial\mathcal{M}} \Phi\, \partial_\perp \Phi \,, \tag{3.1}$$

where $\hat\mu = \mu/\langle\Phi^2\rangle^2$ is a fiducial worldsheet scale and

$$S_D[\Phi] = \frac{1}{4\pi} \oint_{\partial\mathcal{M}} d^2z \, \left(\ln \Phi^2\right)\, R^{(2)} \tag{3.2}$$

is the usual form of the action for the dilaton field in a string sigma-model. In (3.1) we have incorporated an arbitrary constant $\alpha_+$ in the cosmological constant operator so that it may be made marginal (i.e. $\alpha_+$ is adjusted so that $\mathrm{e}^{\alpha_+\varphi}$ has conformal dimension 1) [5]. This term is then to be thought of as being induced by higher-loop effects, in analogy to the situation in two-dimensional quantum gravity [23] whereby quantum fluctuations also change the factor parametrizing the cosmological constant operator. Since $\Phi$ can vary in the directions normal to $\partial\mathcal{M}$ in $\mathcal{M}$, its value in the bulk parametrizes the worldsheet dilaton field $\ln\Phi^2$. The fact that it is constant here owes to the property that the present model induces a theory of pure two-dimensional quantum gravity, i.e. it lives in a zero-dimensional target space. In the next section we will consider the coupling of the theory (3.1) to dynamical matter fields from the bulk point of view and hence the properties of the theory when embedded into higher-dimensional spacetimes, more precisely in flat toroidal backgrounds where again the dilaton field should be constant. In this section we shall use conformal invariance to



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determine a relation between the parameters of the Liouville action in (3.1) and the induced dilaton field.

For this, we parametrize the field $\Phi$ as

$$\Phi(x) = \mathrm{e}^{\gamma\phi(x)}\,, \qquad x \in \mathcal{M} \tag{3.3}$$

In the functional form (3.3) the dilaton field is thus related to the external tachyon operator in target space whose vacuum expectation value is the scale $\hat{\mu}$ of the induced two-dimensional theory. Thus the present framework gives a relationship between the bulk dilaton field and the target space tachyon operator, such that the background tachyon field is responsible for the spontaneous breaking of the three-dimensional conformal symmetry. The constant $\gamma$ will be fixed by demanding that the two-dimensional action (3.1) be conformally-invariant, as it should be due to the Weyl invariance of the bulk theory.

Under a shift of the linear dilaton field,

$$\phi \to \phi + \delta\phi\,, \qquad \text{with } \delta\phi\,\Big|_{\partial\mathcal{M}} = \text{constant}\,, \tag{3.4}$$

corresponding to a global scale transformation of the worldsheet, the action (3.1) changes as

$$\begin{aligned} S_\partial[\varphi;\Phi] \to S_\partial[\varphi;\Phi] &- \frac{2\gamma Q}{\alpha_+}\,\chi_E(\partial\mathcal{M})\,\delta\phi\,+ \\ &+2\kappa\left(1-\mathrm{e}^{2\gamma\delta\phi}\right)\oint_{\partial\mathcal{M}}\Phi\,\partial_\perp\Phi - 2\kappa\gamma\,\mathrm{e}^{2\gamma\delta\phi}\oint_{\partial\mathcal{M}}\Phi^2\,\partial_\perp\delta\phi\,, \end{aligned} \tag{3.5}$$

where $\chi_E(\partial\mathcal{M}) = \frac{1}{4\pi}\oint_{\partial\mathcal{M}} d^2z\,R^{(2)} = 2(1-h_{\partial\mathcal{M}})$ is the Euler character of the Riemann surface $\partial\mathcal{M}$ and $h_{\partial\mathcal{M}}$ is its genus. When the theory (3.1) is coupled to a string sigma-model (as we will do in the next section), the dilatonic shift should be absorbed into a redefinition of the string coupling constant as $g_s \to \mathrm{e}^{\delta\phi}\,g_s$. Since the latter quantity appears in the string perturbation expansion as $g_s^{2(1-h_{\partial\mathcal{M}})}$, this means that, if we assume that $\Phi$ is constant everywhere on $M$, then the linear variation in $\phi$ should be $2(1-h_{\partial\mathcal{M}})\delta\phi$ which leads to the constraint

$$-\frac{2\gamma Q}{\alpha_+} = 1 \tag{3.6}$$

on the parameters of the theory (3.1). However, in (3.5) there is generally an extra conformally non-invariant term which is a remnant from the embedding of the worldsheet $\partial\mathcal{M}$ into the bulk of the three-manifold $\mathcal{M}$. For the time being we shall ignore this extra three-dimensional piece to illustrate how one can fix the parameters. When Liouville gravity is coupled to conformal matter fields of central charge $c \leq 1$, the requirement that the total quantum action be conformally invariant fixes the parameters of the Liouville action as [23]

$$Q = \sqrt{\frac{25-c}{3}}\,, \qquad \alpha_+ = -\frac{Q}{2} + \sqrt{\frac{1-c}{12}}\,. \tag{3.7}$$

This shows that the field $\phi(x)$ in (3.3) can be identified as a string dilaton provided we take

$$\gamma = \frac{1}{4}\left(1 - \sqrt{\frac{1-c}{25-c}}\right). \tag{3.8}$$

## 4. Sigma-model couplings and quantum duality transformations

We will now consider the coupling of deformed Liouville gravity to conformal matter fields. From the point of view of a topological membrane, this means that we add propagating gauge field degrees of freedom to the bulk, in addition to the gravity and scalar fields. As always this is done in a conformally invariant way. The simplest situation is described by the bulk action

$$\mathcal{S}[e,\omega,\Phi,A] = S_{CTMG}[e,\omega;\Phi] + S^{[U(1)]}_{CTMGT}[A;e,\Phi]\,, \tag{4.1}$$

where

$$S^{[U(1)]}_{CTMGT}[A;e,\Phi] = -\frac{1}{4}\int_{\mathcal{M}} d^3x\ \left(\sqrt{g}\ g^{\mu\rho}g^{\nu\sigma}\,\frac{1}{\Phi^2}\,F_{\mu\nu}F_{\rho\sigma} - \frac{2m}{\pi}\,\epsilon^{\mu\nu\lambda}A_\mu\,\partial_\nu A_\lambda\right) \tag{4.2}$$

is the action for $U(1)$ topologically massive gauge theory conformally-coupled to the scalar field $\Phi$. Here $A$ is an abelian gauge connection on $\mathcal{M}$ and $F = dA$ is its field strength. The non-polynomial coupling to the scalar field $\Phi$ ensures the invariance of the action (4.2) under the Weyl transformations (2.8). When $\langle\Phi^2\rangle = 0$ the model (4.2) is equivalent to pure Maxwell theory which has a massless photon. However, when the conformal symmetry is spontaneously broken there is a massive propagating photon in the bulk with topological mass

$$M_p = \frac{m\,\langle\Phi^2\rangle}{\pi}\,. \tag{4.3}$$

When $\partial\mathcal{M} \neq \emptyset$, the only gauge non-invariant term in (4.2) is the Chern-Simons action and therefore we expect the same WZNW model to be induced on the boundary as in the case of a non-conformal bulk coupling. Indeed, because of the Dirichlet boundary condition (2.10), the variation of the action (4.2) restricted to the boundary is

$$\delta S^{[U(1)]}_{CTMGT}[A;e,\Phi]\ \Big|_{\partial\mathcal{M}} = \oint_{\partial\mathcal{M}} \left(\Pi^z\delta A_z + \Pi^{\bar z}\delta A_{\bar z}\right)\,, \tag{4.4}$$

where

$$\Pi^{z,\bar z} = \frac{1}{\Phi^2}\,\sqrt{g}\,F^{z\perp} + \frac{2m}{\pi}\,\sqrt{g}\,g^{z\bar z}A_{\bar z,z} \tag{4.5}$$

is the canonical momentum conjugate to the gauge field. This is the same boundary variation that occurs in the usual case [4, 20] and one can therefore proceed to factorize the gauge-fixed path integral for the gauge theory (4.2) into bulk and boundary components. When the $U(1)$ gauge group is compact, the bulk theory (4.1) induces the two-dimensional boundary action

$$\mathcal{S}_\partial[\varphi,\phi,\theta]=S_\partial[\varphi;\phi]+S_{XY}^{[S^1]}[\theta]\,, \tag{4.6}$$

where

$$S_{XY}^{[S^1]}[\theta]=\frac{m}{2\pi}\oint_{\partial\mathcal{M}}d^2z\;\partial_z\theta\,\partial_{\bar z}\theta \tag{4.7}$$

is the linear sigma-model action with $\theta\in S^1$ the pure gauge part of $A$ on the boundary $\partial\mathcal{M}$. The gauge field Chern-Simons coefficient is related to the radius $R$ of the circle $S^1$ by

$$m=\frac{R^2}{\alpha'}\,. \tag{4.8}$$

Thus the action (4.1) induces a coupling of the deformed Liouville theory (3.1) to the $c=1$ conformal field theory of the $XY$ model.

Let us now consider the behaviour of the theory (4.1) under a $T$-duality transformation $R\to\alpha'/R$ of the target space $S^1$ of the $XY$ model (4.7), which is a symmetry of the two-dimensional quantum field theory. This mapping is equivalent to the transformation

$$m\to m^*=\frac{1}{m} \tag{4.9}$$

of the Chern-Simons coefficient of the bulk theory (4.1). The target space $T$-duality transformation therefore arises from an $S$-duality transformation of the three-dimensional quantum field theory. The effects of this transformation in topologically massive gauge theory have been extensively studied in [24, 25]. Although it is not a precise symmetry of the bulk theory,[4] in the same sense as the way it acts on the boundary theory, it does provide a one-to-one mapping between the spectrum of the quantum gauge theory and its dual, i.e. it preserves the Landau level structure of states. Since in the present case the mass gap (4.3) between states involves the dynamical scalar field $\Phi$, we can demand that it be invariant under the mapping (4.9). More precisely, we introduce a dual scalar field $\Phi^*(x)$ on $\mathcal{M}$ so that

$$M_p=\frac{m\,\langle\Phi^2\rangle}{\pi}=\frac{m^*\langle\Phi^{*\,2}\rangle}{\pi}\,. \tag{4.10}$$

This leads to the $T$-duality transformation law

$$\Phi\to\Phi^*=m\,\Phi \tag{4.11}$$

or, using (3.3) and (3.8), the transformation of the linear dilaton

$$\phi\to\phi^*=\phi+4\ln\frac{R^2}{\alpha'}\,. \tag{4.12}$$

[4] Under the $T$-duality transformation (4.9), the topologically massive gauge theory action (4.2) is mapped into a Chern-Simons-Proca gauge theory with the same mass (4.3). Moreover, the mapping interchanges winding numbers of matter fields and monopole numbers in the spectrum of the quantum gauge theory. See [25] for details.

Eq. (4.12) differs by a factor of $\frac{1}{8}$ from the usual transformation law (1.3) for the dilaton field under $T$-duality. The discrepency can be traced back to the extra local operator that appears in the deformed Liouville action (3.1) which provides a coupling to the bulk degrees of freedom. This means that the scale-invariance arguments which we used to fix the constant $\gamma$ in (3.8) are not precisely valid in the present case (c.f. (3.5)). As we have stressed in sections 2 and 3, the action (3.1) is intrinsically three-dimensional in origin, so that the usual arguments of two-dimensional quantum gravity require some modification that would presumably change the numerical value of $\gamma$ given in section 3 and give agreement with the standard results. In any case, we take (4.12) to be the three-dimensional version of the dilaton transformation law.

In fact, the above arguments lead immediately to an intimate relationship between $T$-duality and $S$-duality. On the worldsheet boundary the $T$-duality and $S$-duality mappings are given, respectively, by

$$\begin{aligned} T\,:\,R &\to \frac{\alpha'}{R}\,, \qquad g_s^2 \to (\mathrm{const.})\cdot g_s^2\,, \\ S\,:\,g_s &\to \frac{1}{g_s} \end{aligned} \tag{4.13}$$

where $g_s$ is the string coupling constant (1.1). On the other hand, in order to keep the mass gap of the bulk three-dimensional theory invariant under the mapping (4.9), the dilaton condensate must change (up to a constant) like $\langle\Phi^2\rangle \to 1/\langle\Phi^2\rangle$. Thus in the bulk the analogs of the $S$-duality and $T$-duality transformations are respectively

$$\begin{aligned} \boldsymbol{S}\,:\,m &\to \frac{1}{m}\,, \qquad \Phi \to m\,\Phi\,, \\ \boldsymbol{T}\,:\,\Phi^2 &\to \frac{\mathrm{const.}}{\Phi^2}\,. \end{aligned} \tag{4.14}$$

Although in the boundary theory the $S$-duality and $T$-duality symmetries appear to be unrelated, the bulk theory unifies them via the change of role of the scalar field $\Phi$ in the bulk-boundary correspondence, i.e. $\boldsymbol{S} \sim T$ and $\boldsymbol{T} \sim S$. Specifically, the duality transformations (4.14) both change the topological masses of the three-dimensional theory (photon mass for $\boldsymbol{S}$ and graviton mass for $\boldsymbol{T}$). This gives a remarkable dynamical equivalence between the quantum geometry of the target space and the non-perturbative properties of the quantum string theory.

Within the present three-dimensional framework there is an interesting generalization of this construction for higher-dimensional toroidal string compactifications. For this, we consider a compact $U(1)^D$ conformally-coupled topologically massive gauge theory with potentials $A^I$ and associated field strengths $F^I = dA^I$, $I = 1,\dots,D$. The action is

$$S_{CTMGT}^{[U(1)^D]}[A;e,\Phi] = -\frac{1}{4}\sum_{I,J}\int_{\mathcal{M}} d^3x\ \left(\sqrt{g}\ g^{\mu\rho}g^{\nu\sigma}\,\frac{1}{\Phi_{IJ}^2}\,F_{\mu\nu}^I F_{\rho\sigma}^J - \frac{2}{\pi}\,K_{IJ}\,\epsilon^{\mu\nu\lambda}A_\mu^I\,\partial_\nu A_\lambda^J\right)\,, \tag{4.15}$$

where $K_{IJ}$ is a non-degenerate constant $D \times D$ matrix, and the functions $\Phi_{IJ}(x) = \Phi_{JI}(x)$ each transform under restricted three-dimensional conformal transformations according to (2.8). The action (4.15) induces the $c = D$ conformal field theory of the $XY$ model

$$S_{XY}^{[T^D]}[\theta] = \frac{1}{2\pi} \sum_{I,J} \oint_{\partial\mathcal{M}} d^2z \ K_{IJ} \, \partial_z \theta^I \, \partial_{\bar{z}} \theta^J \ , \tag{4.16}$$

where the pure gauge degrees of freedom $\theta^I$ of the gauge fields $A^I$ live in a $D$-torus $T^D$. This identifies the Chern-Simons coefficient matrix as

$$K_{IJ} = \frac{1}{\alpha'} \Big( G_{IJ} + B_{IJ} \Big) \, , \tag{4.17}$$

where $G_{IJ}$ and $B_{IJ}$ are the target space graviton and antisymmetric tensor condensates, respectively. In the phase of non-zero meson condensates, it follows from the gauge field equations of motion that the local propagating degrees of freedom of the model (4.15) can be characterized by the mass matrix

$$M_{IJ} = \frac{1}{\pi\alpha'} \sum_L G_{JL} \left\langle \Phi_{IL}^2 \right\rangle \, . \tag{4.18}$$

Here we have used the fact that the bulk part of the Chern-Simons action in (4.15), and hence the gauge field propagator, depends only on the symmetric part of the matrix $K_{IJ}$, i.e. on the metric tensor $G_{IJ}$ of $T^D$. This can be seen via an integration by parts of the Chern-Simons three-form.

A $T$-duality transformation of the quantum field theory (4.16) corresponds to inversion of the Chern-Simons coefficient matrix $K_{IJ} \to (K^{-1})_{IJ}$, or in terms of the metric of $T^D$

$$G_{IJ} \to G_{IJ}^* = \Big[ (K^\top)^{-1} \, G \, K^{-1} \Big]_{IJ} \, . \tag{4.19}$$

Defining a duality transformation $\Phi_{IJ} \to \Phi_{IJ}^*$ such that the mass matrix (4.18) is preserved by (4.19), we find

$$\Phi_{IJ}^2 \to \Phi_{IJ}^{*\,2} = \sum_L \Big[ K \, G^{-1} \, K^\top \, G \Big]_{JL} \Phi_{IL}^2 \, . \tag{4.20}$$

If we now define

$$\begin{aligned} \Phi(x) &= \det_{I,J} \left[ \Phi_{IJ}(x) \right] \\ &= \mathrm{e}^{\phi(x)/4} \, , \end{aligned} \tag{4.21}$$

then (4.20) implies that the linear dilaton field $\phi$ transforms under toroidal $T$-duality transformations as

$$\phi \to \phi^* = \phi + 2 \ln \frac{\det_{I,J} \left[ G_{IJ} \right]}{\det_{I,J} \left[ G_{IJ}^* \right]} \, . \tag{4.22}$$

Modulo the usual factor of $\frac{1}{8}$, (4.22) has the precise form of the dilaton transformation law for generic toroidal compactifications [26].

Eq. (4.20) defines a dilaton transformation law that has no analog in the induced two-dimensional theory and is purely three-dimensional in origin. It comes from the $D > 1$ local gauge field excitations characterized by (4.18). One can think of this generalization as providing an independent dilaton field $\phi_I(x)$ along each direction of the target space, where $\mathrm{e}^{\phi_I(x)/4}$ are the eigenvalues of the symmetric matrix function $\Phi_{IJ}(x)$, such that $\phi_I(x)$ controls the size of the compactified direction $I$. The canonical dilaton in (4.21) is then the average of these fields over the various directions and it mediates the scaling properties of the entire spacetime as a whole. It is a highly non-trivial property of the three-dimensional models that the local dynamics of the bulk theory, measured here by (4.3) and (4.18), conspire to yield the anticipated boundary properties of the induced two-dimensional sigma models. For instance, for the transformation law (4.22) to hold it is crucial that *only* the symmetric part of the Chern-Simons coefficient matrix $K_{IJ}$ appear in (4.18). The present approach thus yields a natural dynamical and geometrical origin for the dilaton in string sigma-models.

## 5. Dynamical string-coupling generation

As we have discussed, the dilaton field of deformed Liouville theory couples to the bulk of the three-manifold $\mathcal{M}$ and is strictly speaking not a constant field in target space. The bulk dynamics of conformally-coupled topologically massive gravity control the worldsheet properties of this field, and this raises the possibility that the scalar field $\Phi$ may in fact generate a dynamical string coupling constant (1.1). The most elegant way of exploring this possibility is to consider the induced two-dimensional theory that arises when, instead of the Dirichlet boundary condition (2.10), one imposes a Neumann boundary condition on the scalar field $\Phi(x)$ on $\partial\mathcal{M}$,

$$\partial_\perp \Phi = 0\,. \tag{5.1}$$

In other words, the field $\Phi$ is constant in a neighbourhood of $\partial\mathcal{M}$ in $\mathcal{M}$ but it is an arbitrary function $\Phi(z,\bar{z})$ on the worldsheet $\partial\mathcal{M}$.[5] Now the conformal symmetry (2.8) in the bulk is broken down to the subgroup of local $SO(2,1)$ conformal transformations which are constant along the directions normal to $\partial\mathcal{M}$ in $\mathcal{M}$. Such a restriction on the local conformal symmetry group of the three-dimensional theory is very natural from the point of view of the fact that it should induce the local scale invariance of the two-dimensional boundary theory. Moreover, the additional boundary term in (2.9) vanishes with the choice of Neumann boundary condition.

[5] A more mathematical way of saying this is that $\Phi$ defines a global section of the normal bundle over $\partial\mathcal{M}$ in $\mathcal{M}$.



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However, there are now extra boundary terms associated with the non-constant field $\Phi|_{\partial\mathcal{M}}$ coming from the additional boundary terms in (2.19) and (2.21) due to the non-vanishing variation $\delta\Phi$ on $\partial\mathcal{M}$. The simplest way to incorporate such terms is to consider a boundary Weyl transformation of the action (2.19). From (2.5) and (2.23) it follows that the boundary components of the spin-connection transform as

$$\omega_z \to \omega_z + \frac{\gamma}{2}\tau_3\,\partial_{\bar z}\phi\,, \qquad \omega_{\bar z} \to \omega_{\bar z} - \frac{\gamma}{2}\tau_3\,\partial_z\phi\,. \tag{5.2}$$

It follows that the effect of allowing the field $\phi$ to vary along the boundary is to shift the diagonal components $\omega^3_{z,\bar z}$ of the spin-connection. The action (2.19) is therefore modified by the shift (5.2) to

$$\begin{aligned} S_T[e,\omega,\phi,\beta] = S_{\rm bulk}[e,\omega,\phi,\beta] + \oint_{\partial\mathcal{M}} d^2z\,\Big\{ {\rm e}^{2\gamma\phi}\ {\rm tr}(e_{\bar z}\beta_z - \kappa e_z\omega_{\bar z}) - \\ -\frac{k}{8\pi}\left[\omega^+_z\omega^-_{\bar z} + \omega^+_{\bar z}\omega^-_z + 2\left(\omega^3_z + \frac{\gamma}{2}\partial_{\bar z}\phi\right)\left(\omega^3_{\bar z} - \frac{\gamma}{2}\partial_z\phi\right)\right]\Big\}\,. \end{aligned} \tag{5.3}$$

The additional contributions involving the dilaton field in (5.3) are most transparent when written in terms of the worldsheet $T$-dual field $\widetilde\phi$ defined by

$$\partial_z\phi = \partial_{\bar z}\widetilde\phi\,, \qquad \partial_{\bar z}\phi = -\partial_z\widetilde\phi\,. \tag{5.4}$$

The dual dilaton field $\widetilde\phi$ is only locally defined on the boundary $\partial\mathcal{M}$ according to the Poincaré lemma. Substituting (5.4) into (5.3), reflecting the anti-holomorphic diagonal component $\omega^3_{\bar z} \to -\omega^3_{\bar z}$, and integrating the $\omega^3\,d\widetilde\phi$ cross-term by parts (ignoring the singularities in the definition of $\widetilde\phi$), we arrive at

$$\begin{aligned} S_T[e,\omega,\phi,\beta] = S_{\rm bulk}[e,\omega,\phi,\beta] + \oint_{\partial\mathcal{M}} d^2z\,\Big\{ {\rm e}^{2\gamma\phi}\ {\rm tr}(e_{\bar z}\beta_z - \kappa e_z\omega_{\bar z}) \\ -\frac{k}{8\pi}\left[\omega^+_z\omega^-_{\bar z} + \omega^+_{\bar z}\omega^-_z - 2\omega^3_z\omega^3_{\bar z} - \gamma\,\widetilde\phi\,R_{z\bar z} + \frac{\gamma^2}{2}\partial_z\widetilde\phi\,\partial_{\bar z}\widetilde\phi\right]\Big\}\,, \end{aligned} \tag{5.5}$$

where $R_{z\bar z} = \partial_z\omega^3_{\bar z} - \partial_{\bar z}\omega^3_z$. The advantage of this dual representation is that the scalar field $\widetilde\phi$ couples in a conformally-invariant way to the induced worldsheet geometry.

The shift (5.2) also produces a coupling of $\widetilde\phi$ to the Liouville field which comes from the gauged WZNW action (2.21) (see (2.34)). After the rescaling $\varphi \to \varphi/\sqrt{10k}$, we find that the boundary action (2.37) acquires the additional term

$$\frac{\gamma}{20\pi\alpha_+}\oint_{\partial\mathcal{M}} d^2z\,\left(-\frac{\gamma}{2\alpha_+}\,\partial_z\widetilde\phi\,\partial_{\bar z}\widetilde\phi + \frac{1}{\alpha_+}\,\widetilde\phi\,R^{(2)} + \partial_z\widetilde\phi\,\partial_{\bar z}\varphi + \partial_{\bar z}\widetilde\phi\,\partial_z\varphi\right)\,, \tag{5.6}$$

where we have defined $k = 2/5\alpha_+^2$. Furthermore, when one shifts the Liouville field $\varphi$ to write the cosmological constant operator of the Liouville part of the total action in standard form, there are additional kinetic terms induced for the dilaton field $\phi$. The total boundary action in this case is thus

$$\begin{aligned} S_\partial[\varphi;\phi,\widetilde{\phi}] = \oint_{\partial\mathcal{M}} d^2z \ \left(\frac{1}{8\pi}\,\partial_z\varphi\,\partial_{\bar z}\varphi + \frac{Q}{8\pi}\,\varphi\,R^{(2)} + \frac{\rho^2\,\hat\mu}{8\pi\alpha_+^2}\,\mathrm{e}^{\alpha_+\varphi}\right) - \\ - \frac{2\gamma Q}{\alpha_+}\,S_D[\phi] + \frac{\gamma}{5\alpha_+^2}\,S_D[\widetilde{\phi}] + \\ + \oint_{\partial\mathcal{M}} d^2z \ \left[\frac{2\gamma^2}{\pi\alpha_+^2}\,\partial_z\phi\,\partial_{\bar z}\phi - \frac{\gamma}{2\pi\alpha_+}\Big(\partial_z\phi\,\partial_{\bar z}\varphi + \partial_{\bar z}\phi\,\partial_z\varphi\Big)\right] + \\ + \frac{\gamma}{20\pi\alpha_+}\oint_{\partial\mathcal{M}} d^2z \ \left(-\frac{\gamma}{2\alpha_+}\,\partial_z\widetilde{\phi}\,\partial_{\bar z}\widetilde{\phi} + \partial_z\widetilde{\phi}\,\partial_{\bar z}\varphi + \partial_{\bar z}\widetilde{\phi}\,\partial_z\varphi\right) - \\ - \frac{\gamma^2}{5\pi\alpha_+^2}\oint_{\partial\mathcal{M}} d^2z \ \Big(\partial_z\phi\,\partial_{\bar z}\widetilde{\phi} + \partial_{\bar z}\phi\,\partial_z\widetilde{\phi}\Big)\,, \end{aligned} \tag{5.7}$$

where $S_D[\phi] = \frac{1}{4\pi}\oint_{\partial\mathcal{M}} d^2z\ \phi\,R^{(2)}$ is the usual dilaton action.

In the resulting path integral the bulk and boundary degrees of freedom are now completely decoupled and the action (5.7) defines a purely two-dimensional field theory (in contrast to the previous case). We may think of this action as defining the local dynamics of a string-coupling field $g_s(z,\bar z)$. This field couples to the two-dimensional worldsheet quantum gravity and its action (5.7) involves the corresponding (singular) worldsheet dual field. In fact, the action (5.7) is invariant under the worldsheet duality transformation

$$\phi \to -\frac{\alpha_+}{10Q}\widetilde{\phi}\,, \tag{5.8}$$

provided that the parameters of the Liouville part of the total action are fixed as

$$Q = \alpha_+ = -\frac{2}{5}\,. \tag{5.9}$$

Thus in the phase (5.9) of unbroken worldsheet $T$-duality symmetry, the action (5.7) describes a new form of non-unitary matter fields coupled to two-dimensional quantum gravity. It would be interesting to explore if this phase is related to the strong-coupling phase of Liouville theory whose properties are largely unknown.

It is in this way that the bulk dynamics of topologically massive gravity can induce a dynamical scale parameter which, when coupled to string sigma-models as described in section 4, allows one to dynamically control the size of the radii of the compactified dimensions of the target space. Via its coupling to $\varphi$ in (5.7), it also controls the extra "time" direction induced by the Liouville field. The creation of the field $\Phi(z,\bar z)$ thereby give an induced theory which is more general than string theory, i.e. it puts the topological membrane approach into a more unified setting like $M$-theory [19] or other extensions of string theory. In particular, since the theory (5.7) involves both the field $\phi$ and its (independent) dual $\widetilde{\phi}$, the induced worldsheet model appears to yield two extra dimensions in target space and may play a role in understanding the extra dimensionality of $F$-theory [27].

Moreover, if we interpret the appearence of both $\phi$ and $\widetilde{\phi}$ as implying the existence of two independent Liouville fields, and hence two "times", then the construction of this section can be thought of as giving a worldsheet origin, via topological membranes, for models with two time evolution parameters [28] which have symmetry groups that coincide with those of $AdS_D$ spacetimes. The model (5.7) therefore also suggests a natural dynamical and geometrical origin, in terms of topological membranes, for the believed correspondence between conformal field theories and supergravity on anti-de Sitter spacetimes [29].

It would be interesting to exploit properties of the worldsheet theory (5.7), such as its worldsheet $T$-duality symmetry, to explore features of the spacetime in connection with $M$-theory and these other generalizations. In fact, the present construction can be thought of as giving a dynamical origin to the appearence of extra dimensions in this framework and as illustrating how the dynamics of Liouville gravity appear in the 11-dimensional model. In this framework these dynamical components are all described by basic string degrees of freedom which are induced by the bulk dynamics of the topological membrane, thereby illustrating the relevance of both string dynamics and topological membranes to the full dynamics of $M$-theory. It would be most interesting to see what this implies for a target space Lagrangian formalism for the latter theory.

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